Integration: Geometry Translations

To translate a figure in the direction described by an ordered pair, add the ordered pair to the coordinates of each vertex of the figure.

**Example**

The vertices of $\triangle ABC$ are $A(-2, 2)$, $B(-1, -2)$, and $C(-6, 1)$. Graph the triangle. Then graph the triangle after a translation 7 units right and 3 units up.

$A(-2, 2) + (7, 3) \rightarrow A'(5, 5)$
$B(-1, -2) + (7, 3) \rightarrow B'(6, 1)$
$C(-6, 1) + (7, 3) \rightarrow C'(1, 4)$.

The vertices of the translated figure are $A'(5, 5)$, $B'(6, 1)$, and $C'(1, 4)$.

Graph $\triangle ABC$ and $\triangle A'B'C'$.

Find the coordinates of the vertices of each figure after the translation described. Then graph the figure and its translation.

1. $\triangle XYZ$ with vertices $X(-1, 2)$, $Y(2, 3)$, and $Z(3, -1)$, translated by $(-2, -3)$

2. polygon $KLMN$ with vertices $K(-1, 1)$, $L(-3, 0)$, $M(-2, -3)$, $N(0, -2)$, translated by $(4, 3)$

3. $\triangle DEF$ with vertices $D(0, 5)$, $E(-1, 3)$, and $F(-3, 4)$, translated by $(2, -1)$

4. pentagon $ABCD$ with vertices $A(4, -1)$, $B(3, 2)$, $C(1, 4)$, $D(-2, 1)$, and $E(-3, -3)$, translated by $(-2, 1)$

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Integration: Geometry
Translations

Name the coordinates of the ordered pair needed to translate each point A to point B.

1. \( A(4, 8) \)
   \( B(8, 2) \)
   \((-4, -6)\) (This is a translation 4 units right and 6 units down)

2. \( A(40, 20) \)
   \( B(220, 140) \)

3. Translate \( \triangle ABC \) with vertices \( A(-1, 4), B(0, 0) \), and \( C(2, 3) \) by \((5, -2)\). Then graph \( \triangle A'B'C' \).
   
   \( A' (4, 2) \)
   \( B' (5, -2) \)
   \( C' (7, 1) \)

4. Rectangle \( QRST \) has vertices \( Q(-1, -2), R(-2, 1), S(4, 3) \), and \( T(5, 0) \). Find the coordinates of the vertices of \( Q'R'S'T' \) after a translation described by \((1, -2)\).

5. The coordinates of the vertices of \( \triangle ABC \)
   are \( A(3, -1), B(0, 2) \) and \( C(3, -2) \). Find the coordinates of the vertices of \( \triangle A'B'C' \), which is \( \triangle ABC \) translated by \((-3, -2)\). Then graph \( \triangle ABC \) and its translation.
   
   \( A' (0, -3) \)
   \( B' (-3, 0) \)
   \( C' (0, -4) \)

6. Square \( ABCD \) has vertex \( A(-5, -12) \). When translated, \( A' \) has coordinates \((6, 10)\). Describe the translation using an ordered pair.
Integration: Geometry
Reflections

When a figure is reflected on a coordinate plane, every point of the figure has a corresponding point on the other side of the line of symmetry.

To reflect a figure over the x-axis, use the same x-coordinate and multiply the y-coordinate by -1.

To reflect a figure over the y-axis, multiply the x-coordinate by -1 and use the same y-coordinate.

Example

$\triangle ABC$ has vertices $A(-2, -2), B(-5, -4), C(-1, -5)$.

$\triangle ABC$ reflected over the x-axis
has vertices at $(-2, 2), (-5, 4), (-1, 5)$.

$\triangle ABC$ reflected over the y-axis
has vertices at $(2, -2), (5, -4), (1, -5)$.

Graph trapezoid BIRD with vertices $B(1, 1), I(2, 4), R(6, 4), and D(7, 1)$.

1. Find the coordinates of the vertices after a reflection over the x-axis.
   Graph the reflection.
   
   $B'$ $(1, -1)$
   $I'$ $(2, -4)$
   $R'$ $(6, -4)$
   $D'$ $(7, -1)$

2. Find the coordinates of the vertices after a reflection over the y-axis.
   Graph the reflection.
   
   $B''(-1, 1)$
   $I''(-2, 4)$
   $R''(-6, 4)$
   $D''(-7, 1)$

Graph parallelogram JUNE with vertices $J(2, -2), U(6, -2), N(8, -5), and E(4, -5)$.

3. Find the coordinates of the vertices after a reflection over the x-axis.
   Graph the reflection.
   
   $J'(2, 2)$
   $U'(6, 2)$
   $N'(8, 5)$
   $E'(4, 5)$

4. Find the coordinates of the vertices after a reflection over the y-axis.
   Graph the reflection.
   
   $J''(-2, -2)$
   $U''(-6, -2)$
   $N''(-8, -5)$
   $E''(-4, -5)$
Integration: Geometry
Reflections

Name the line of symmetry for each pair of figures.

1. \[ X \text{ is the line of symmetry} \]

2. \[ U \text{ is the line of symmetry} \]

3. \[ U \text{ is the line of symmetry} \]

4. Graph \( \triangle BAT \) with vertices \( B(1, 1), A(2, 3), \) and \( T(5, 3) \).
   a. Reflect \( \triangle BAT \) over the \( x \)-axis.
   \[ B'(1, -1), A'(2, -3), T(5, 3) \]
   b. Reflect \( \triangle BAT \) over the \( y \)-axis.
   \[ B''(-1, 1), A''(-2, 3), T''(-5, 3) \]

5. Graph parallelogram \( KENT \) with vertices \( K(1, 2), E(5, 4), N(7, 3), \) and \( T(3, 1) \).
   a. Find the coordinates of the vertices after a reflection over the \( y \)-axis.
   \[ K'(-1, 2), N'(-7, 3), E'(-5, 4), T'(-3, 1) \]
   b. Graph the parallelogram \( K'E'NT' \).

6. Graph \( \triangle USA \) with vertices \( U(0, 4), S(4, 4), \) and \( A(4, 0) \).
   a. Reflect \( \triangle USA \) over the \( y \)-axis, and label \( U'S'A' \).
   \[ U'(0, -4), S'(-4, 4), A'(4, 0) \]
   b. On the same coordinate plane, reflect \( \triangle USA \) over the \( x \)-axis.
   \[ U''(0, -4), S''(-4, -4), A''(4, 0) \]
   c. On the same coordinate plane, reflect \( U'S'A' \) over the \( x \)-axis.
   \[ U'''(0, -4), S'''(-4, -4), A'''(4, 0) \]
   d. Write a statement describing the final appearance of the four graphs.
Integration: Geometry Rotations

Triangle \( XYZ \) has vertices \( X(-4, 1) \), \( Y(-1, 5) \), and \( Z(-6, 9) \).

To rotate \( \triangle XYZ \) 180°, multiply each coordinate by \(-1\).

\((-4, 1) \rightarrow (4, -1)\)
\((-1, 5) \rightarrow (1, -5)\)
\((-6, 9) \rightarrow (6, -9)\)

To rotate \( \triangle XYZ \) 90° counterclockwise, switch the coordinates and multiply the first by \(-1\).

\((-4, 1) \rightarrow (-1, -4)\)
\((-1, 5) \rightarrow (-5, -1)\)
\((-6, 9) \rightarrow (-9, -6)\)

Triangle \( RST \) has vertices \( R(-2, -1) \), \( S(0, -4) \), and \( T(-4, -7) \).

1. Graph \( \triangle RST \).
2. Find the coordinates of the vertices after a 90° counterclockwise rotation. Graph the rotation.

3. Find the coordinates of the vertices after a 180° rotation. Graph the

Rectangle \( TWIN \) has vertices \( T(2, 1) \), \( W(6, 3) \), \( I(5, 5) \), and \( N(1, 3) \).

4. Graph rectangle \( TWIN \).
5. Find the coordinates of the vertices after a 90° counterclockwise rotation. Graph the rotation.

6. Find the coordinates of the vertices after a 180° rotation. Graph the rotation.
Integration: Geometry
Rotations

Determine whether each pair of figures represents a rotation. Write yes or no.

1. Yes
2. No
3. Yes

4. Rectangle WORK has vertices W(1, 3), O(4, 6), R(6, 4), and K(3, 1).
   a. Graph WORK.
   b. Rotate the rectangle 90° counterclockwise, and graph W'O'R'K'.
   c. Rotate the rectangle 180°, and graph W''O''R''K''.

5. Examine the figure at the right.
   a. Does the figure have rotational symmetry?
   b. If so, find the degree turns that show this symmetry.

6. Quadrilateral NEAL has vertices N(3, 5), E(4, 4), A(3, 2), and L(1, 3).
   a. Graph quadrilateral NEAL and its 90° counterclockwise rotation N'E'A'L'.
   b. Rotate N'E'A'L' 90° counterclockwise.
   c. Rotate quadrilateral NEAL 180°. Explain the result.

Two 90° counterclockwise rotations equal one 180° counterclockwise rotation. b. and c. are the same.

7. A triangle is rotated 90° counterclockwise. The coordinates of the vertices of the rotated triangle are (3, 2), (−1, 3), and (2, −3). What are the coordinates of the original triangle? The coordinates of the original triangle are (2, −3), (3, 1), and (−3, −2).